Electrostatic Fields and Field Stress Control ‘Introduction’

- Let’s look at a simple geometry includes infinite parallel plane metal electrodes, which are separated by a distance y and contains two dielectrics ε_{r1} in thickness y_1 and ε_{r2} in thickness y_2.
- We define a vector quantity related to the electrical flux, which is called electrical flux density, *D*. It is the quantity of electrical flux passing through a unit area.

$$D = \psi / A, \quad \psi : \text{electric flux generated by a charge } q$$

- The electrical field *E* is related to flux density *D* by the following mathematical relationship:

$$D = \varepsilon_0 \varepsilon_r \ E$$

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Electrostatic Fields and Field Stress Control ‘Introduction’

- The flux density will be the same in each section of the dielectric:

$$D_1 = D_2 = D_0$$

- The electric field will change at the boundary:

$$E_1 = \frac{1}{\varepsilon_0 \varepsilon_{r1}} D_1 = \frac{1}{\varepsilon_0 \varepsilon_{r1}} D_0$$

$$E_2 = \frac{1}{\varepsilon_0 \varepsilon_{r2}} D_2 = \frac{1}{\varepsilon_0 \varepsilon_{r2}} D_0$$

Electrostatic Fields and Field Stress Control
'Introduction'

- The voltage across dielectric 1 and 2 are:

$$V_1 = \int E.dl = E_1 y_1 = \frac{1}{\varepsilon_0 \varepsilon_{r1}} D_0 y_1$$

$$V_2 = \int E.dl = E_2 y_2 = \frac{1}{\varepsilon_0 \varepsilon_{r2}} D_0 y_2$$

Electrostatic Fields and Field Stress Control ‘Introduction’

- The total voltage, *V*:

$$V = V_1 + V_2 = \frac{1}{\varepsilon_0 \varepsilon_{r1}} D_0 y_1 + \frac{1}{\varepsilon_0 \varepsilon_{r2}} D_0 y_2$$

$$V = D_0 \left[\frac{1}{\varepsilon_0 \varepsilon_{r1}} y_1 + \frac{1}{\varepsilon_0 \varepsilon_{r2}} y_2 \right]$$

$$D_0 = \frac{V}{\frac{1}{\varepsilon_0 \varepsilon_{r1}} y_1 + \frac{1}{\varepsilon_0 \varepsilon_{r2}} y_2}$$

Electrostatic Fields and Field Stress Control 'Introduction'

- We can now find E_1 and E_2:

$$E_1 = \frac{1}{\varepsilon_0 \varepsilon_{r1}} D_0 = \frac{1}{\varepsilon_0 \varepsilon_{r1}} \left(\frac{V}{\left(1/\varepsilon_0 \varepsilon_{r1}\right) y_1 + \left(1/\varepsilon_0 \varepsilon_{r2}\right) y_2} \right)$$

$$E_2 = \frac{1}{\varepsilon_0 \varepsilon_{r2}} D_0 = \frac{1}{\varepsilon_0 \varepsilon_{r2}} \left(\frac{V}{\left(1/\varepsilon_0 \varepsilon_{r1}\right) y_1 + \left(1/\varepsilon_0 \varepsilon_{r2}\right) y_2} \right)$$

Electrostatic Fields and Field Stress Control
‘Introduction’

- <u>Implications</u>
  - In the geometry we considered
    - Uniform flux density
    - Difference in dielectric constant
  - Electric field is higher in the dielectric with lower relative permittivity
  - Electric field is lower in the dielectric with higher relative permittivity

Example

- Calculate the values of electric field in a system where we have two infinite electrodes separated by a distance 1 mm with a voltage of a 1 kV across electrodes. Assume that a slab of dielectric with $\varepsilon_r = 3$ of thickness 0.9 mm is present in the air gap between the electrodes.

Electrostatic Fields and Field Stress Control
‘Introduction’

$$E_1 = \frac{1}{\varepsilon_0 \varepsilon_{r1}} D_0, \quad E_2 = \frac{1}{\varepsilon_0 \varepsilon_{r2}} D_0$$

$$V_1 = E_1 y_1 = \frac{1}{\varepsilon_0 \varepsilon_{r1}} D_0 y_1, \quad V_2 = E_2 y_2 = \frac{1}{\varepsilon_0 \varepsilon_{r2}} D_0 y_2$$

$$V = V_1 + V_2 = \frac{1}{\varepsilon_0 \varepsilon_{r1}} D_0 y_1 + \frac{1}{\varepsilon_0 \varepsilon_{r2}} D_0 y_2 = \frac{D_0}{\varepsilon_0} \left[\frac{1}{\varepsilon_{r1}} y_1 + \frac{1}{\varepsilon_{r2}} y_2 \right]$$

Electrostatic Fields and Field Stress Control ‘Introduction’

$$\frac{D_0}{\varepsilon_0} = \frac{V}{\frac{1}{\varepsilon_{r1}} y_1 + \frac{1}{\varepsilon_{r2}} y_2} = \frac{1}{\frac{1}{1} \times 0.1 + \frac{1}{3} \times 0.9} = 25\,\text{kV/cm}$$

$$E_1 = \frac{1}{\varepsilon_0 \varepsilon_{r1}} D_0 = \frac{25}{1} = 25\,\text{kV/cm},$$

$$E_2 = \frac{1}{\varepsilon_0 \varepsilon_{r2}} D_0 = \frac{25}{3} = 8.33\,\text{kV/cm}$$

Electrostatic Fields and Field Stress Control

‘Electrostatic stresses in single core cable’

- The dielectric material surrounds the conductor and we know that every dielectric material has certain dielectric strength which, if exceeded, will result in rupture of the dielectric.
- In general, the disruptive failure can be prevented by designing the cable such that the maximum electric stress (which occurs at the surface of the conductor) is below that required for short time puncture of dielectric.

Electrostatic Fields and Field Stress Control
‘Electrostatic stresses in single core cable’

- In case the potential gradient is taken a low value, the overall size of the cable above 11 kV becomes relatively large.
- Also, if the gradient is taken large to reduce the overall size of the cable, the dielectric losses increases very much which may result in thermal breakdown of the cable.
- So, a compromise between the two has to be made and normally the value of working stress is taken about 1/5 of the breakdown value for the design purposes.

Electrostatic Fields and Field Stress Control
'Electrostatic stresses in single core cable'

- Assume that we have a cable with an inner conductor radius *r* and outer conductor radius *R*. (coaxial metal cylinders, infinite long)
- The area that flux passes through it increases as we move from inner to outer cylinder.
- Area per unit length $A(r') = 2\pi r'$.
- Therefore, the flux density is given by

$$D = \frac{\psi}{A(r')} = \frac{\psi}{2\pi r' l} = \frac{K_0}{r'}; \quad K_0 = \frac{\psi}{2\pi l}$$

Electrostatic Fields and Field Stress Control
‘Electrostatic stresses in single core cable’

- We can calculate the field or gradient behavior

$$E(r') = g(r') = \frac{D(r')}{\varepsilon_0 \varepsilon_r} = \frac{K_0}{\varepsilon_0 \varepsilon_r r'} = \frac{K}{r'}; \quad K = \frac{K_0}{\varepsilon_0 \varepsilon_r}$$

- Note that if we have a single dielectric the field distribution will not be affected. Depending on the geometry of our system, the magnitude and direction of our field can change with position.
- The total voltage, *V*:

$$V = \int E.dl = \int_r^R E(r')dr' = \int_r^R \frac{K}{r'} dr' = K \ln\left(\frac{R}{r}\right) \Rightarrow K = \frac{V}{\ln\left(R/r\right)}$$

Electrostatic Fields and Field Stress Control
‘Electrostatic stresses in single core cable’

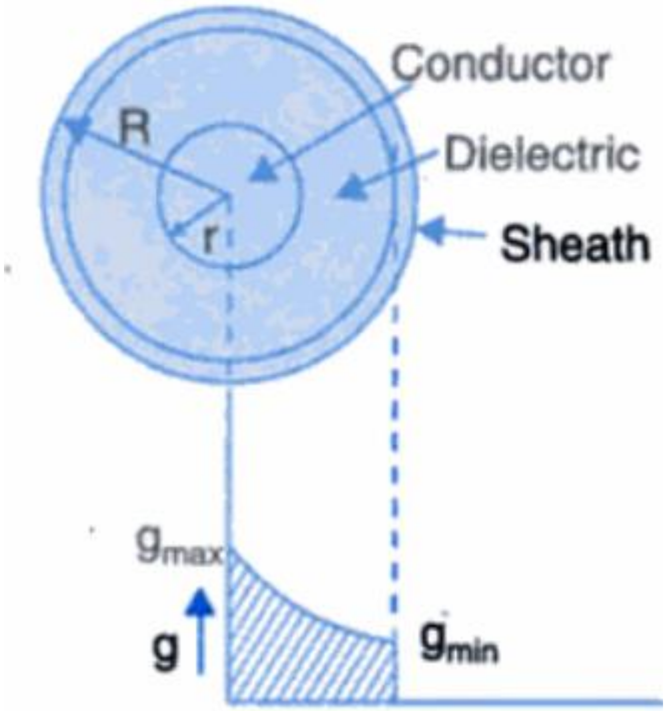
- Therefore the field is given by:

$$E(r') = g(r') == \frac{V}{r' \ln\left(R/r\right)}$$

- It is clear that the gradient is maximum when *r*' = *r* at the surface of the conductor, and is minimum at the inner radius of the sheath

$$E_{\max} = g_{\max} = E(r) = \frac{V}{r \ln\left(R/r\right)},$$

$$E_{\min} = g_{\min} = E(R) = \frac{V}{R \ln\left(R/r\right)}$$



Electrostatic Fields and Field Stress Control
'Electrostatic stresses in single core cable'

- In order to keep a fixed overall size of the cable (R) for a particular operating voltage, there is a particular value of the radius of the conductor, which gives minimum gradient at the surface of the conductor. The objective here is to find the minimum value of g_{max}:

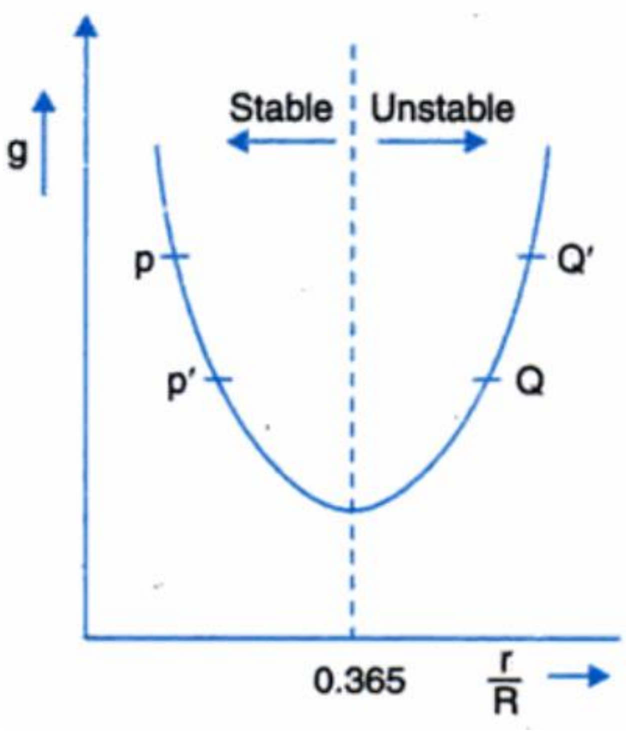
$$f(r) = r \ln\left(\frac{R}{r}\right)$$

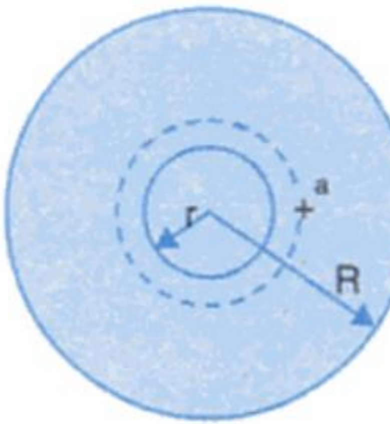
$$\frac{df(r)}{dr} = \ln\left(\frac{R}{r}\right) + r\left(\frac{-R}{r^2}\right)\frac{r}{R} = 0$$

$$\Rightarrow \ln\left(\frac{R}{r}\right) = 1 \Rightarrow \frac{R}{r} = e$$

Electrostatic Fields and Field Stress Control
‘Electrostatic stresses in single core cable’

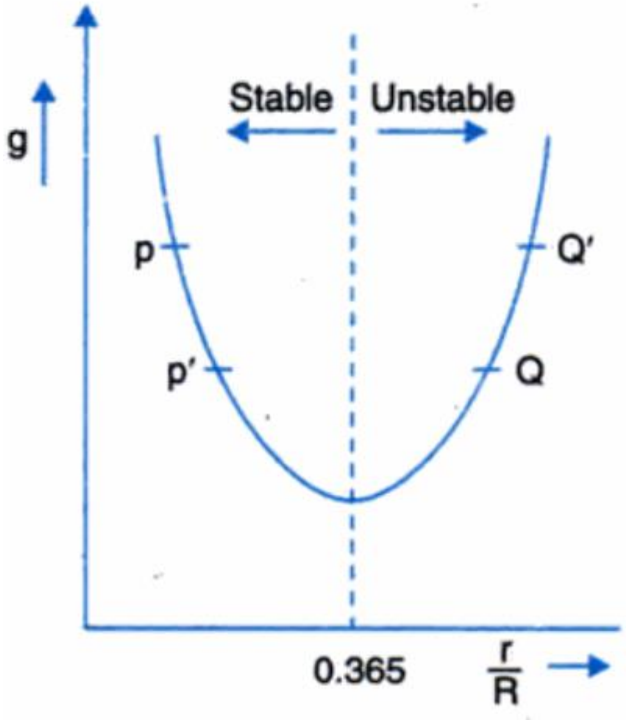
- Stable operation of the cable for particular ratios of r/R.
- What ratio of r/R leads to stable operation of the cable and what ratios will lead to unstable operation?
- Say the ratio corresponds to point Q. Now, due to some manufacturing defects say a thin film of air surrounding the conductor is trapped.
- Let the thickness of this film be a.

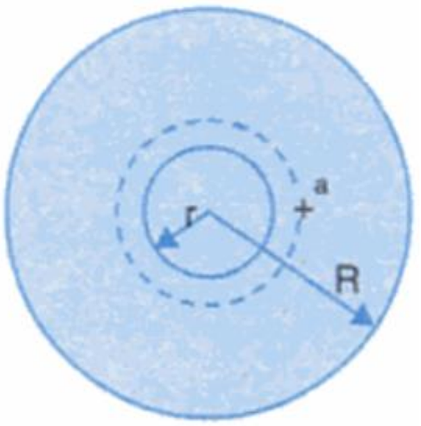




Electrostatic Fields and Field Stress Control
‘Electrostatic stresses in single core cable’

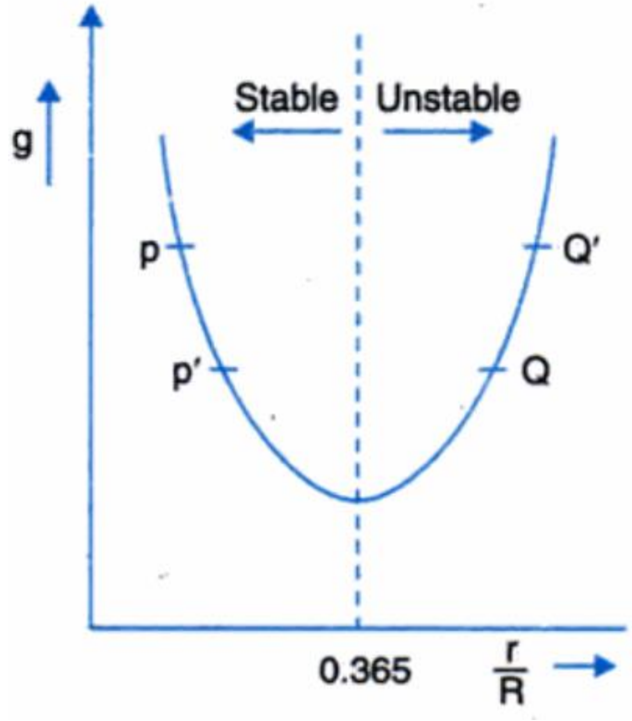
- Since the dielectric strength of the insulating material is taken about 40-50 kV/cm, the air surrounding the conductor is stressed and get ironized.
- Therefore, the effective radius of the conductor will be now $r + a$ and the ratio will be $(r + a)/R$.
- The operating point now shifts to Q'.
- The stress on the dielectric material will increase and this may finally lead to rupture of the material

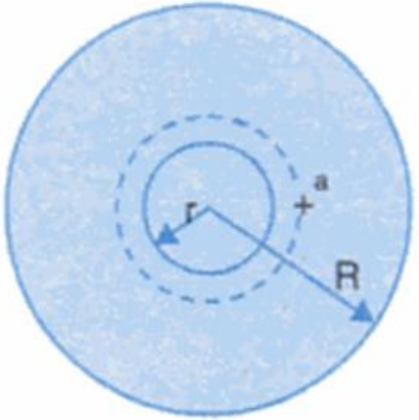




Electrostatic Fields and Field Stress Control
‘Electrostatic stresses in single core cable’

- Let now take a cable with a ratio of r/R such that it corresponds to point P.
- Now, due to some manufacturing defects say a thin film of air surrounding the conductor is trapped.
- Let the thickness of this film be a.





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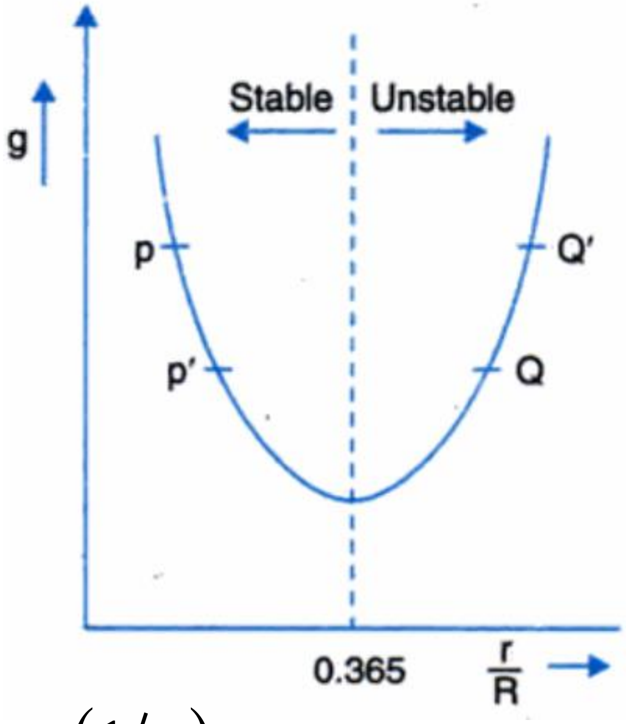
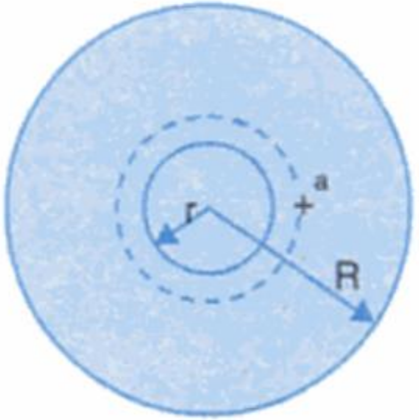
Electrostatic Fields and Field Stress Control
'Electrostatic stresses in single core cable'

- The air surrounding the conductor is stressed and get ironized.
- Therefore, the effective radius of the conductor will be now $r + a$ and the ratio will be $(r + a)/R$.
- The operating point now shifts to P'.
- The cable operate satisfactorily
- The satisfactory condition is: $(r/R) < (1/e)$





Electrostatic Fields and Field Stress Control
‘Electrostatic stresses in single core cable’

<u>Example</u>

- Determine the economic overall diameter of a single core cable metal sheathed for a working voltage of 85 kV is the dielectric strength of the insulating material is 65 kV/cm.

Electrostatic Fields and Field Stress Control
‘Electrostatic stresses in single core cable’

$$E_{\max} = g_{\max} = E(r) = \frac{V}{r\ln(R/r)} = \frac{V}{r}$$

$$r = \frac{V}{g_{\max}} = \frac{85}{65} = 1.3 \text{ cm}$$

Diameter of conductor = 2.6 cm

Diameter of sheath = $2.6e = 7.07$ cm

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Electrostatic Fields and Field Stress Control 'Electrostatic stresses in single core cable'

- The field or gradient is given by:

$$E(r') = g(r') = \frac{V}{r'\ln\left(R/r\right)}$$

- Maximum and average fields or gradients, E_{max} and E_{avg}:

$$E_{avg} = \frac{1}{R-r}\int_r^R E(r')dr' = \frac{1}{R-r}\int_r^R \frac{V}{r\ln\left(R/r\right)}dr' = \frac{1}{R-r}\frac{V}{\ln\left(R/r\right)}\int_a^c \frac{1}{r'}dr' = \frac{V}{R-r}$$

$$E_{\max} = E(a) = \frac{V}{r\ln\left(R/r\right)}$$

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Electrostatic Fields and Field Stress Control
'Electrostatic stresses in single core cable'

- **Filed inhomogeneity or non uniformity factor**, η: It is defined as the ratio of the average field to the maximum field. It gives an indication about the efficiency of insulation used in the system.

$$\eta = \frac{E_{avg}}{E_{\max}} = \frac{\left(\frac{V}{R-r}\right)}{\left(\frac{V}{R\ln(R/r)}\right)} = \frac{r\ln(R/r)}{R-r}$$

Electrostatic Fields and Field Stress Control ‘Electrostatic stresses in single core cable’

- We will consider 3 situations:
  - r = 10 m, r = 5 cm, r = 1 mm,
  - Thickness is 10 cm,
  - System voltage = 100 kV
- Recall the field equation:

$$E(x) = \frac{V}{x \ln\left(R / r\right)}$$

Electrostatic Fields and Field Stress Control
'Electrostatic stresses in single core cable'

- Case 1: r = 10 m, R = 10.1 m

$$E(r) = \frac{V}{r\ln\left(R/r\right)} = \frac{100}{10\times\ln\left(10.1/10\right)} = 1.005\ MV/m$$

$$E(R) = \frac{V}{R\ln\left(R/r\right)} = \frac{100}{10.1\times\ln\left(10.1/10\right)} = 0.994\ MV/m$$

$$E_{avg} = \frac{V}{R-r} = \frac{100}{10} = 1\ MV/m$$

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Electrostatic Fields and Field Stress Control
‘Electrostatic stresses in single core cable’

- Case 2: r = 5 cm, R = 15 cm

$$E(r)=\frac{V}{r\ln\left(R/r\right)}=\frac{100}{5\times\ln\left(15/5\right)}=1.82\ MV/m$$

$$E(R)=\frac{V}{R\ln\left(R/r\right)}=\frac{100}{15\times\ln\left(15/5\right)}=0.606\ MV/m$$

$$E_{avg}=\frac{V}{R-r}=\frac{100}{10}=1\ MV/m$$

Electrostatic Fields and Field Stress Control
‘Electrostatic stresses in single core cable’

- Case 3: r = 1 mm, R = 101 mm

$$E(r)=\frac{V}{r\ln\left(R/r\right)}=\frac{100}{1\times\ln\left(101/1\right)}=21.7\ MV/m$$

$$E(R)=\frac{V}{R\ln\left(R/r\right)}=\frac{100}{101\times\ln\left(101/1\right)}=0.215\ MV/m$$

$$E_{avg}=\frac{V}{R-r}=\frac{100}{10}=1\ MV/m$$

Electrostatic Fields and Field Stress Control
‘Electrostatic stresses in single core cable’

- <u>General behavior:</u>
- At *r* becomes smaller, the field becomes more inhomogeneous: higher values close to the inner conductor and lower values close to the outer conductor.
- Note that the integral of *E* must be the same in all cases as *V* is fixed.
- Average field is unchanged. However, the insulator may be stressed above its breakdown strength close to the inner conductor as radius decreases.

r (m)	E_{max} (MV/m)	E_{min} (MV/m)	E_{avg} (MV/m)	η
0.001	21.7	0.215	1	0.046
0.05	1.82	0.606	1	0.545
10	1.005	0.994	1	0.996

Electrostatic Fields and Field Stress Control
'Electrostatic stresses in single core cable'

Example

- Assume that we have a cable with inner conductor radius of 0.9 cm and an insulation thickness of 0.7 cm. if the breakdown strength of the insulation is 12 kV/mm, what is the maximum voltage that can be applied across the cable and then calculate the non-uniformity factor.

Electrostatic Fields and Field Stress Control
‘Electrostatic stresses in single core cable’

$$E(r) = \frac{K}{r};$$

$$V = \int E.dl = \int_a^c E(r)dr = \int_a^c \frac{K}{r}dr = K\ln\left(\frac{c}{a}\right) \Rightarrow K = \frac{V}{\ln(c/a)}$$

$$E(r) = \frac{V}{r\ln(c/a)}$$

Electrostatic Fields and Field Stress Control
'Electrostatic stresses in single core cable'

$$E(r) = \frac{V}{r\ln(c/a)}$$

$$E_{\max} = E(a) = \frac{V}{a\ln(c/a)}$$

$$E_{\max} = 12\,\text{kV/mm} = 120\,\text{kV/cm} = \frac{V}{0.9\ln(1.6/0.9)}$$

$$V = 62.2\,\text{kV}$$

Electrostatic Fields and Field Stress Control
‘Electrostatic stresses in single core cable’

$$\eta = \frac{E_{avg}}{E_{\max}}$$

$$E_{avg} = \frac{1}{c-a}\int_a^c E(r)dr = \frac{1}{c-a}\int_a^c \frac{V}{r\ln(c/a)}dr = \frac{1}{c-a}\frac{V}{\ln(c/a)}\int_a^c \frac{1}{r}dr = \frac{V}{c-a}$$

$$E_{\max} = E(a) = \frac{V}{a\ln(c/a)}$$

Electrostatic Fields and Field Stress Control
'Electrostatic stresses in single core cable'

$$\eta = \frac{E_{avg}}{E_{\max}} = \frac{\left(\dfrac{V}{c-a}\right)}{\left(\dfrac{V}{a\ln(c/a)}\right)} = \frac{a\ln(c/a)}{c-a} = \frac{0.9\ln(1.6/0.9)}{0.7} = 0.74$$

Electrostatic Fields and Field Stress Control
‘Grading of high voltage cables’

- Grading means the distribution of dielectric material such that the difference between the maximum gradient and minimum gradient is reduced.
- The cable of the same size could be operated at higher voltages, or for the same operating voltage a cable of relatively smaller size could be used.
- Methods of grading
  - ➢Capacitance grading where more than one dielectric material is used
  - ➢Intersheath grading where the same dielectric material is used, but potentials at certain radii are held to certain values by interposing thin metal sheaths

Electrostatic Fields and Field Stress Control 'Grading of high voltage cables, capacitance grading'

- If we have one single dielectric material, the gradient at any radius x will be:

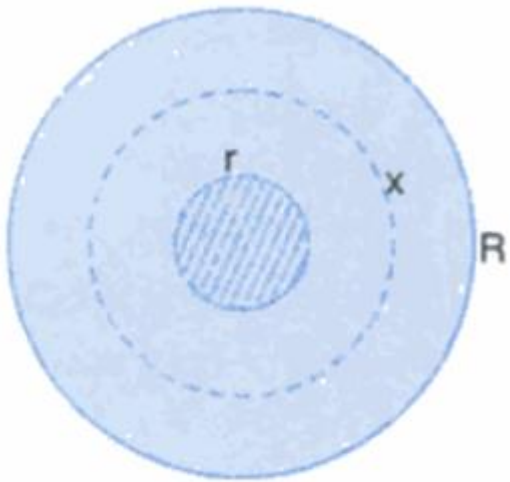
$$E(x) = \frac{K_0}{\varepsilon x}$$

- If we could use an infinite number of materials with varying permittivities given by

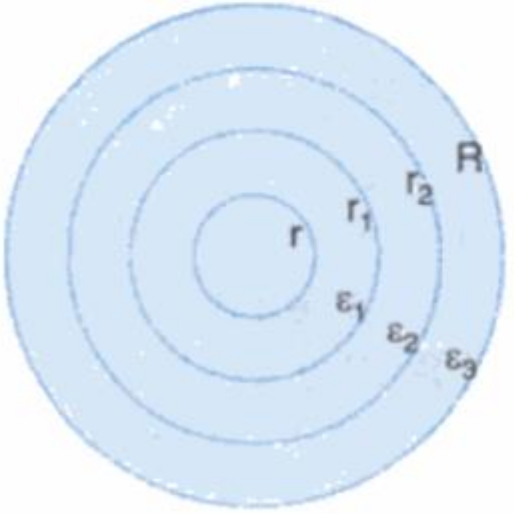
$$\varepsilon = \frac{k}{x},$$

then, the gradient at any radius x becomes constant

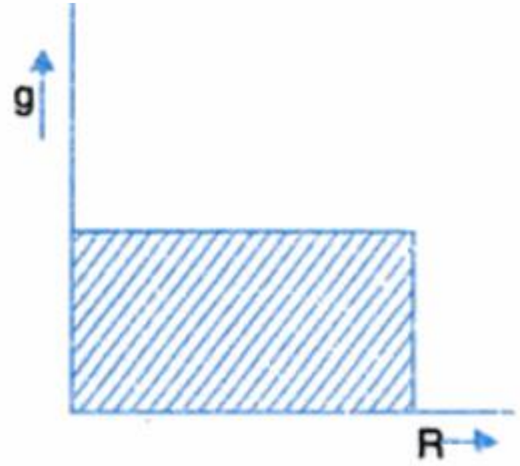
- This is impossible, but normally 2 or 3 materials are used



Electrostatic Fields and Field Stress Control
‘Grading of high voltage cables, capacitance grading’



- Let there be 3 materials placed at radii r, r_1, and r_2, respectively.
- Let the dielectric strength and working stresses of this material be G_1, G_2, G_3, and g_1 , g_2 and g_3, respectively.
- There are 2 possibilities:
  1. The F.O.S, f, for all materials be the same, therefore the working stress of various materials different.
  2. The same working stress for different materials.



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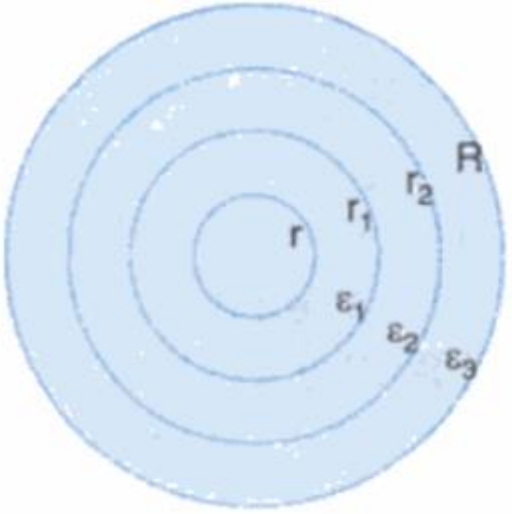
Electrostatic Fields and Field Stress Control ‘Grading of high voltage cables, capacitance grading’

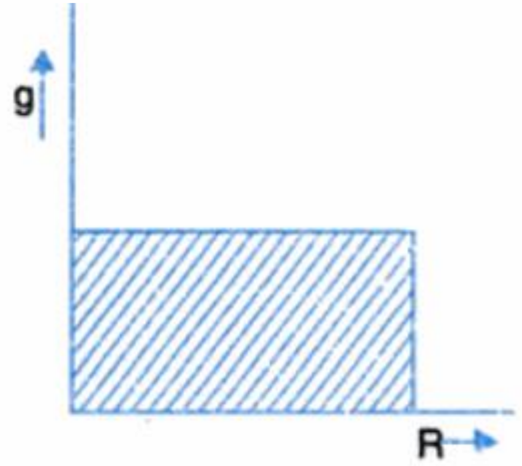
i) The gradients at r, r_1, and r_2,will be

$$g_1 = \frac{K_0}{\varepsilon_1 r} = \frac{G_1}{f}$$

$$g_2 = \frac{K_0}{\varepsilon_2 r_1} = \frac{G_2}{f}$$

$$g_3 = \frac{K_0}{\varepsilon_3 r_2} = \frac{G_2}{f}$$





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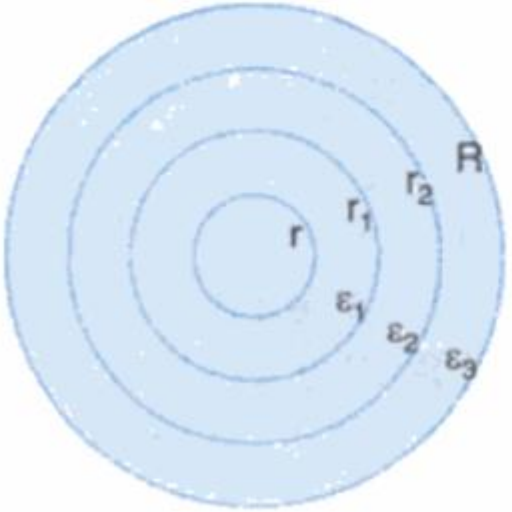
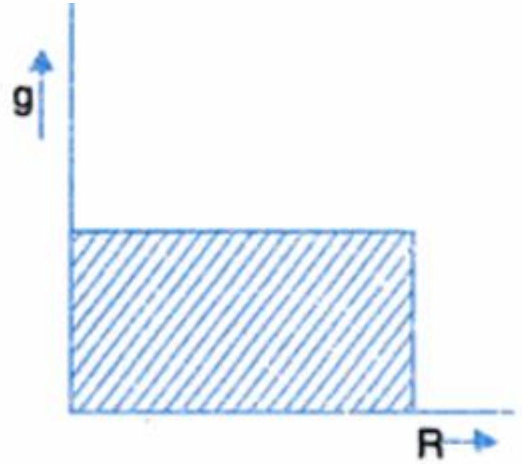
Electrostatic Fields and Field Stress Control ‘Grading of high voltage cables, capacitance grading’

$$\varepsilon_1 r G_1 = \varepsilon_2 r_1 G_2 = \varepsilon_3 r_2 G_3$$

$$r < r_1 < r_2 \Rightarrow \varepsilon_1 G_1 > \varepsilon_2 G_2 > \varepsilon_3 G_3$$

- This means the material with highest product of dielectric strength and permittivity should be placed nearest to the conductor and the other layers should be in the descending order of the product of dielectric strength and permittivity.





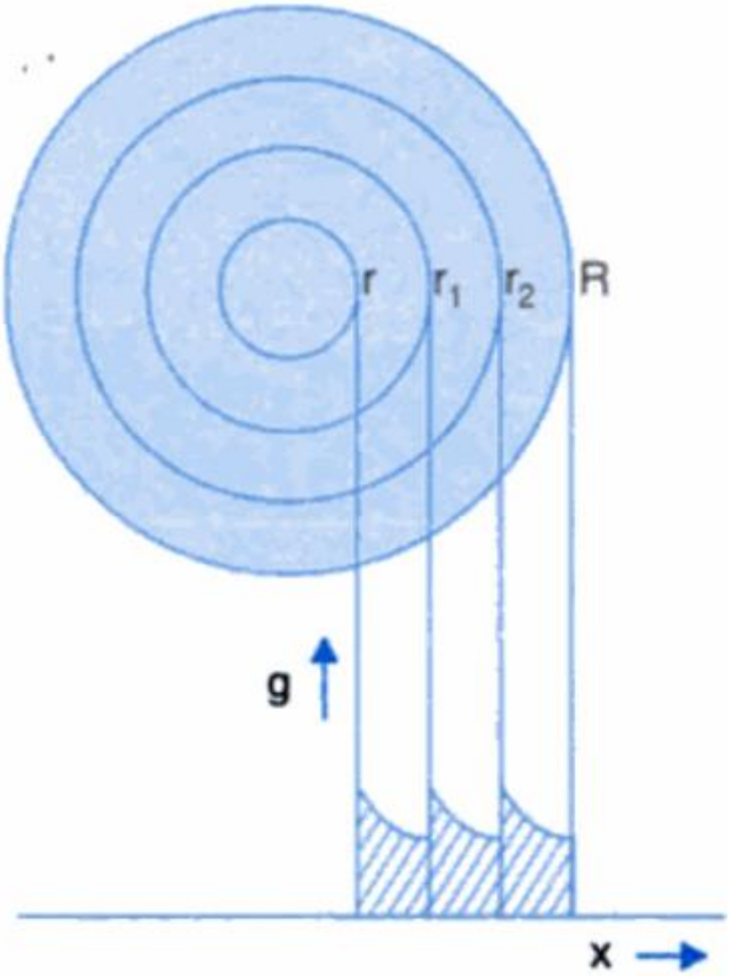
Electrostatic Fields and Field Stress Control
‘Grading of high voltage cables, capacitance grading’

ii) The gradient at r, r_1, and r_2, will be

$$g = \frac{K_0}{\varepsilon_1 r} = \frac{K_0}{\varepsilon_2 r_1} = \frac{K_0}{\varepsilon_3 r_2}$$

$$\varepsilon_1 r = \varepsilon_2 r_1 = \varepsilon_3 r_2 \Rightarrow r < r_1 < r_2 \Rightarrow \varepsilon_1 > \varepsilon_2 > \varepsilon_3$$

- This means the material with highest permittivity should be placed nearest to the conductor and the other layers should be in the descending order of their permittivities.



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Electrostatic Fields and Field Stress Control ‘Grading of high voltage cables, capacitance grading’

- The total voltage, *V*:

$$V=\int_{r}^{r_1}E(r')dr'+\int_{r_1}^{r_2}E(r')dr'+\int_{r_2}^{R}E(r')dr'$$

$$=\int_{r}^{r_1}\frac{K_0}{\varepsilon_1 r'}dr'+\int_{r_1}^{r_2}\frac{K_0}{\varepsilon_2 r'}dr'+\int_{r_2}^{R}\frac{K_0}{\varepsilon_3 r}dr'$$

$$=g_{max}r\ln\left(\frac{r_1}{r}\right)+g_{max}r_1\ln\left(\frac{r_2}{r_1}\right)+g_{max}r_2\ln\left(\frac{R}{r_2}\right)$$

$$=g_{max}\left[r\ln\left(\frac{r_1}{r}\right)+r_1\ln\left(\frac{r_2}{r_1}\right)+r_2\ln\left(\frac{R}{r_2}\right)\right]$$

Electrostatic Fields and Field Stress Control ‘Grading of high voltage cables, capacitance grading’

Example

- A single core covered cable is to be designed for 66 kV to earth. Its conductor radius is 0.5 cm and its 3 insulating materials A, B, and C have relative permittivities of 4, 2.5, and 4 with maximum permissible stresses of 50, 30 and 40 kV/cm, respectively. Determine the minimum internal diameter of the lead sheath. Discuss the arrangement of the insulating materials.

Electrostatic Fields and Field Stress Control ‘Grading of high voltage cables, capacitance grading’

$$\varepsilon_A G_A = 4 \times 50 = 200$$

$$\varepsilon_B G_B = 2.5 \times 30 = 75$$

$$\varepsilon_C G_C = 4 \times 40 = 160$$

$$\varepsilon_A G_A > \varepsilon_C G_C > \varepsilon_B G_B$$

$$\varepsilon_1 = 4$$

$$\varepsilon_2 = 4$$

$$\varepsilon_3 = 2.5$$

Electrostatic Fields and Field Stress Control ‘Grading of high voltage cables, capacitance grading’

$$g_1 = \frac{K_0}{\varepsilon_1 r} = \frac{G_1}{f}$$

$$g_2 = \frac{K_0}{\varepsilon_2 r_1} = \frac{G_2}{f}$$

$$g_3 = \frac{K_0}{\varepsilon_3 r_2} = \frac{G_2}{f}$$

$$\varepsilon_1 r G_1 = \varepsilon_2 r_1 G_2 \Rightarrow r_1 = 0.625 \text{ cm}$$

$$\varepsilon_1 r G_1 = \varepsilon_3 r_2 G_3 \Rightarrow r_2 = 1.330 \text{ cm}$$

Electrostatic Fields and Field Stress Control ‘Grading of high voltage cables, capacitance grading’

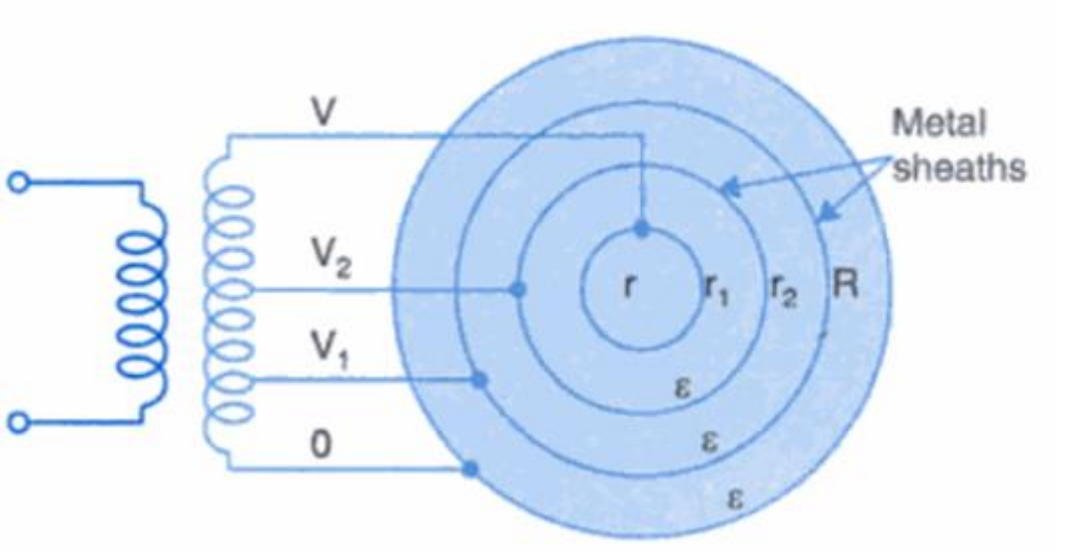
$$V = \int_{r}^{r_1} E(r')dr'\,{}_{1} + \int_{r_1}^{r_2} E(r')dr' + \int_{r_2}^{R} E(r')dr'$$

$$= \int_{r}^{r_1} \frac{K_0}{\varepsilon_1 r'} dr'\,{}_{1} + \int_{r_1}^{r_2} \frac{K_0}{\varepsilon_2 r'} dr' + \int_{r_2}^{R} \frac{K_0}{\varepsilon_3 r} dr'$$

$$= g_1 r \ln\left(\frac{r_1}{r}\right) + g_2 r_1 \ln\left(\frac{r_2}{r_1}\right) + g_3 r_2 \ln\left(\frac{R}{r_2}\right) \Rightarrow D = 2R = 7.53 \text{ cm}$$

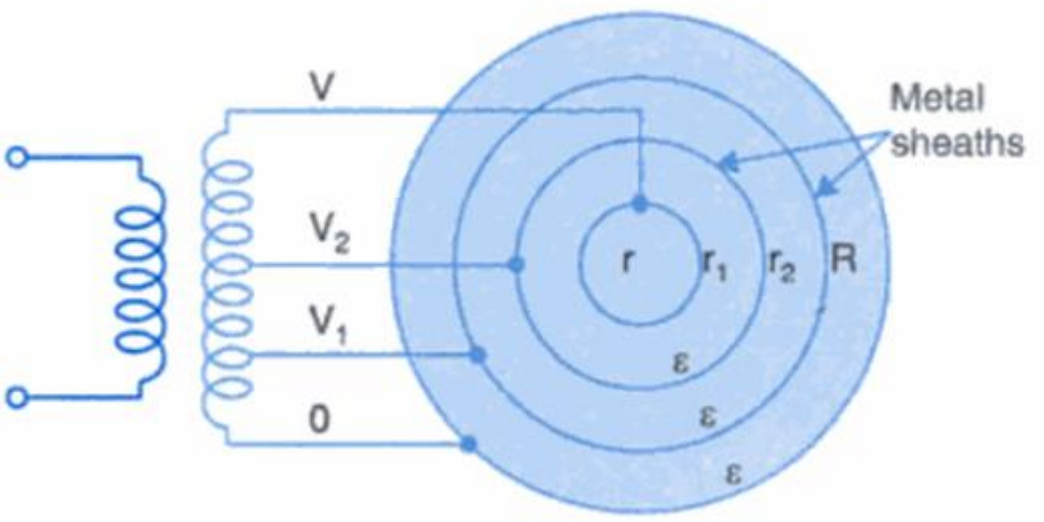
Electrostatic Fields and Field Stress Control ‘Grading of high voltage cables, intersheath grading’

- An auxiliary transformer is used to maintain the metal sheath and the power conductor at certain potentials; thereby the stress distribution is forced to be different from one which it would be without the intersheaths.



Electrostatic Fields and Field Stress Control ‘Grading of high voltage cables, intersheath grading’

- The objective is to show that the gradient with intersheath will be smaller than the gradient without intersheath for the same overall radius and the same operating voltage.
- Since a homogenous material is being used, the maximum value of the stress at various intersheaths is the same.



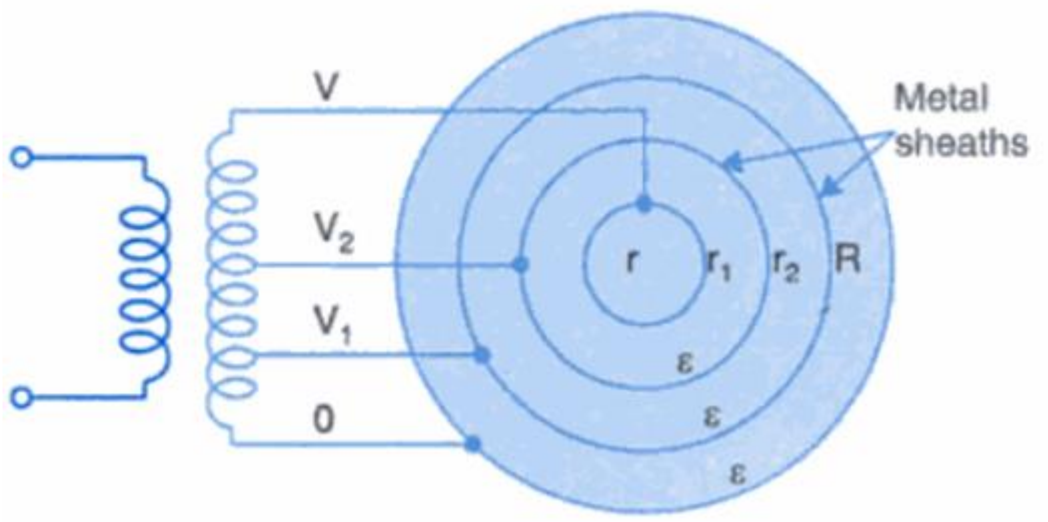
Electrostatic Fields and Field Stress Control ‘Grading of high voltage cables, intersheath grading’

- Let the thickness of the materials be such that

$$\frac{r_1}{r} = \frac{r_2}{r_1} = \frac{R}{r_2} = \alpha$$

- With this arrangement, the gradient at the surface of the conductor

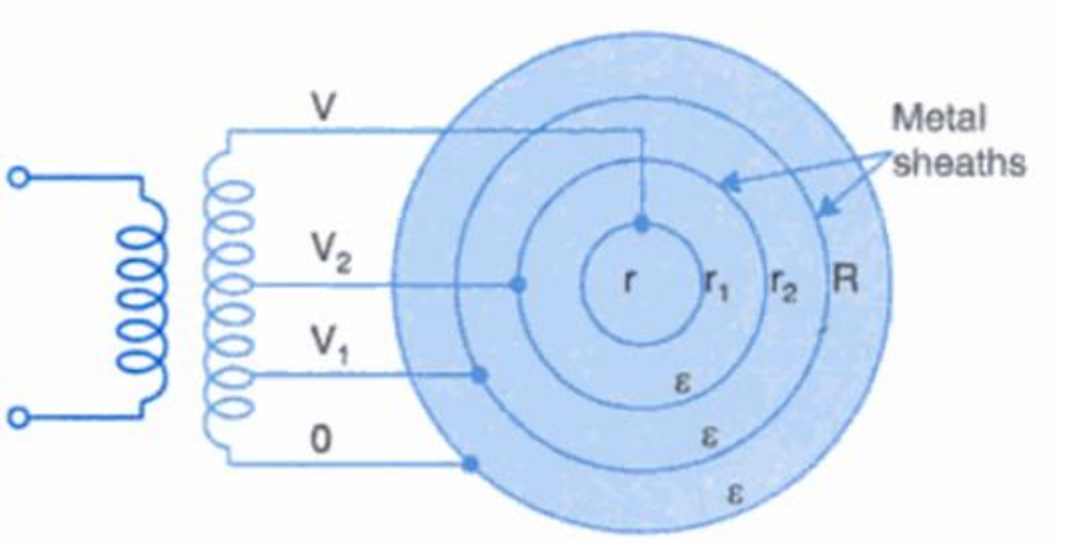
$$g_{\max} = \frac{V - V_2}{r \ln\left(r_1 / r\right)}$$



Electrostatic Fields and Field Stress Control ‘Grading of high voltage cables, intersheath grading’

- Similarly, the gradients at radii r_1 and r_2 respectively are

$$\frac{V_2 - V_1}{r_1 \ln\left(r_2 / r_1\right)} \text{ and } \frac{V_1}{r_2 \ln\left(R / r_2\right)}$$



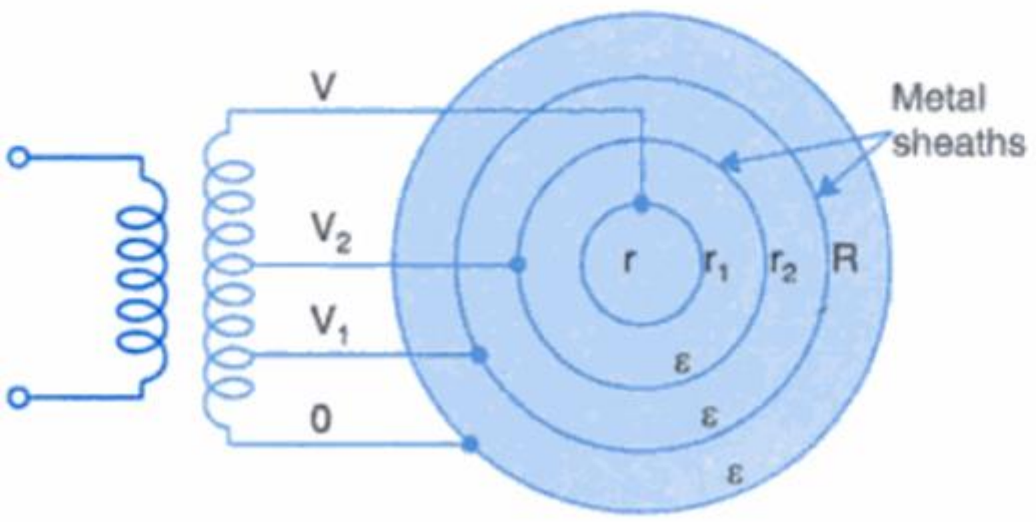
Electrostatic Fields and Field Stress Control ‘Grading of high voltage cables, intersheath grading’

- Since g_{max} is the same at various radii

$$\frac{V-V_2}{r\ln\left(r_1/r\right)}=\frac{V_2-V_1}{r_1\ln\left(r_2/r_1\right)}=\frac{V_1}{r_2\ln\left(R/r_2\right)}$$

$$\frac{V-V_2}{r\ln\left(\alpha\right)}=\frac{V_2-V_1}{r_1\ln\left(\alpha\right)}=\frac{V_1}{r_2\ln\left(\alpha\right)}$$

$$\frac{V-V_2}{r}=\frac{V_2-V_1}{r_1}=\frac{V_1}{r_2}$$



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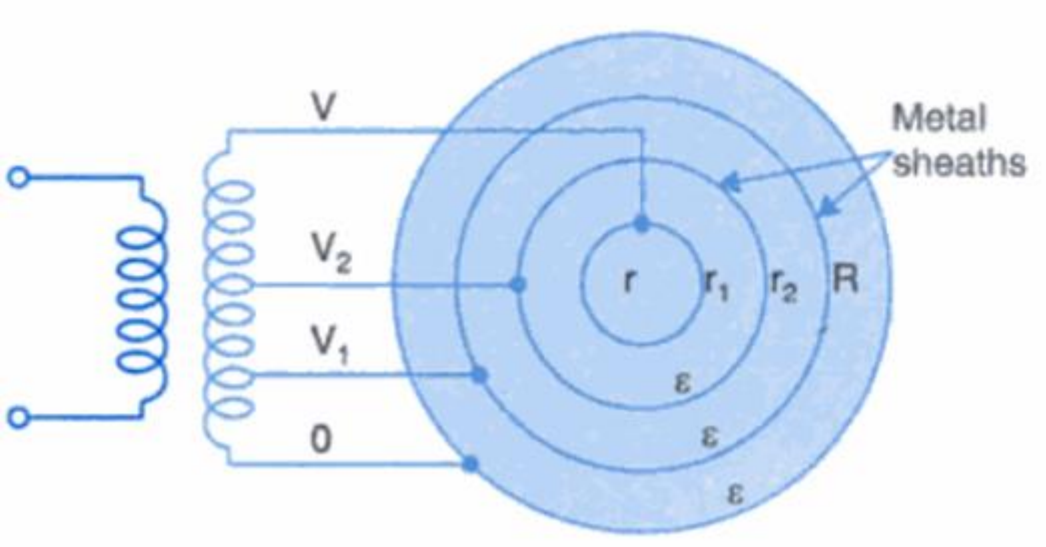
Electrostatic Fields and Field Stress Control ‘Grading of high voltage cables, intersheath grading’

- V_1 in terms of V_2

$$\frac{V_2 - V_1}{r_1} = \frac{V_1}{r_2} \Rightarrow \frac{V_2}{r_1} - \frac{V_1}{r_1} = \frac{V_1}{r_2}$$

$$\frac{V_2}{r_1} = V_1\left(\frac{1}{r_2} + \frac{1}{r_1}\right) \Rightarrow V_2 = V_1\left(\frac{r_1}{r_2} + 1\right)$$

$$V_2 = V_1\left(\frac{1}{\alpha} + 1\right) \Rightarrow V_1 = V_2\left(\frac{\alpha}{\alpha + 1}\right)$$

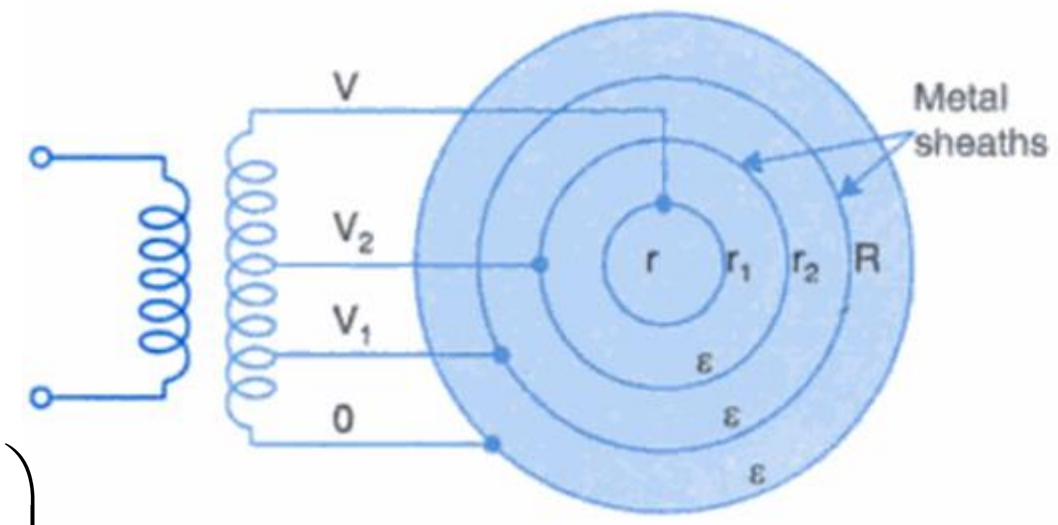


- V_2 in terms of V

$$\frac{V-V_2}{r}=\frac{V_2-V_1}{r_1}\Rightarrow V-V_2=\frac{V_2-V_1}{\alpha}$$

$$V-V_2=\frac{V_2-V_2\left(\dfrac{\alpha}{\alpha+1}\right)}{\alpha}\Rightarrow V-V_2=V_2\left(\frac{1}{\alpha^2+\alpha}\right)$$

$$V=V_2\left(1+\frac{1}{\alpha^2+\alpha}\right)\Rightarrow V_2=V\left(\frac{\alpha^2+\alpha}{\alpha^2+\alpha+1}\right)$$

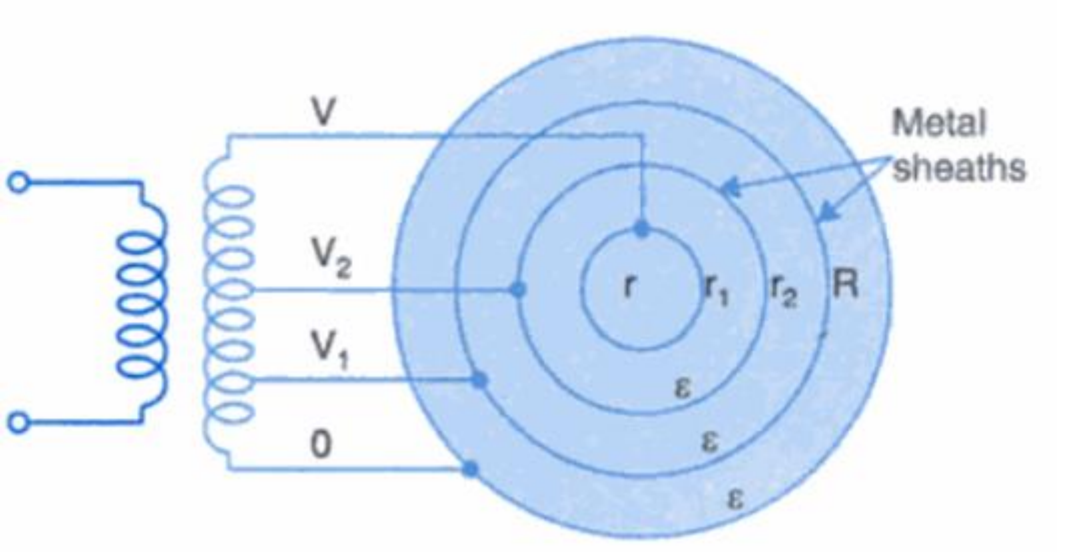


Electrostatic Fields and Field Stress Control 'Grading of high voltage cables, intersheath grading'

- Now $g_{\max}$ is given by

$$g_{\max} = \frac{V - V_2}{r\ln(\alpha)} = \frac{V - V\left(\dfrac{\alpha^2 + \alpha}{\alpha^2 + \alpha + 1}\right)}{r\ln(\alpha)}$$

$$= \frac{V}{r\ln(\alpha)}\left(\frac{1}{\alpha^2 + \alpha + 1}\right)$$



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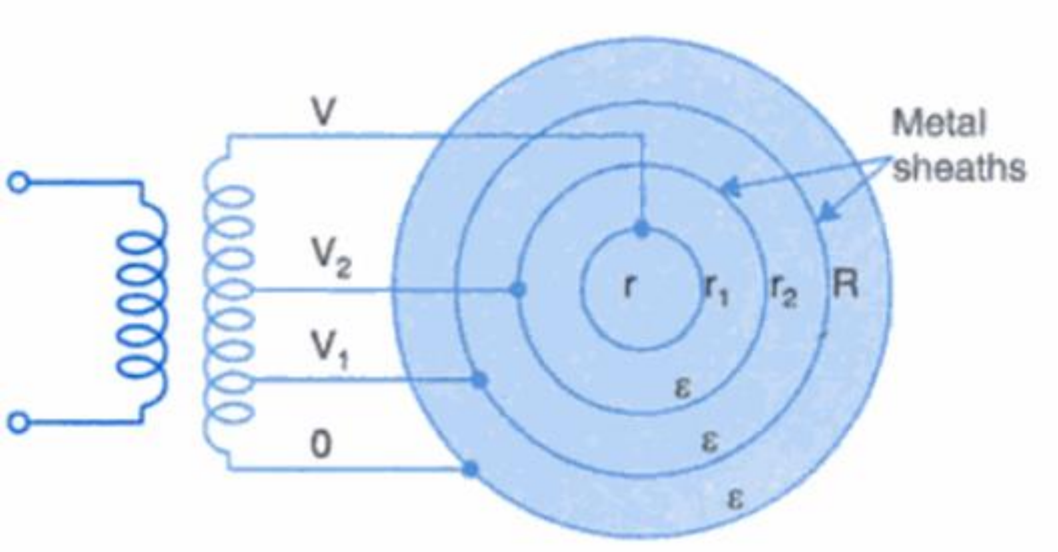
- Now, the gradient at the surface of the conductor without intersheath

$$g = \frac{V}{r\ln(R/r)} = \frac{V}{3r\ln(\alpha)}$$

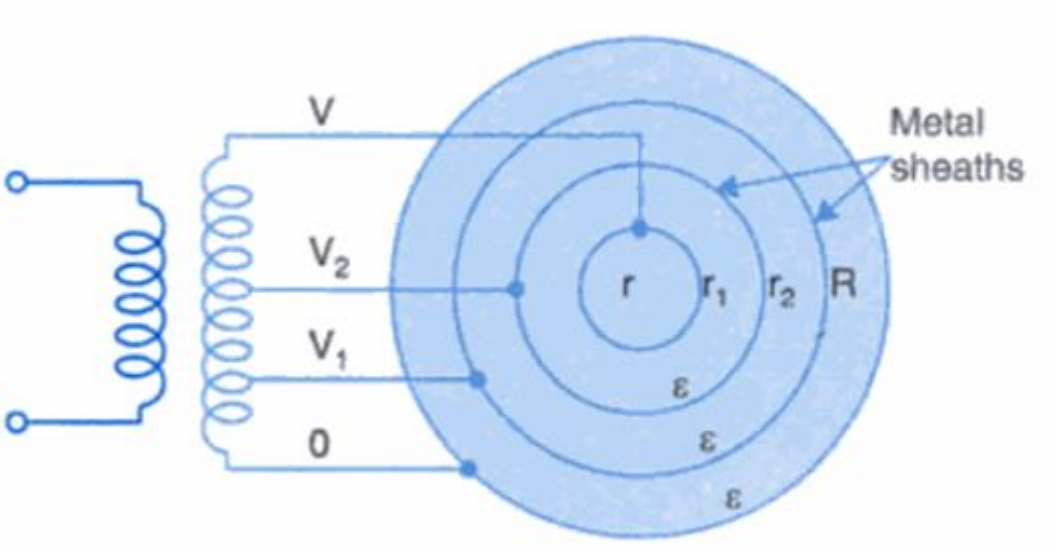


- Therefore,

$$\frac{g_{max}}{g} = \frac{3}{\alpha^2+\alpha+1} < 1 \Rightarrow g_{max} < g$$

Electrostatic Fields and Field Stress Control ‘Grading of high voltage cables, intersheath grading’

- Gradient with intersheath is lower than without intersheath for the same overall size and operating voltage of the cable.
- This means a cable of a particular size can be operated for higher voltages or for a particular voltage the size of the cable can be reduced.

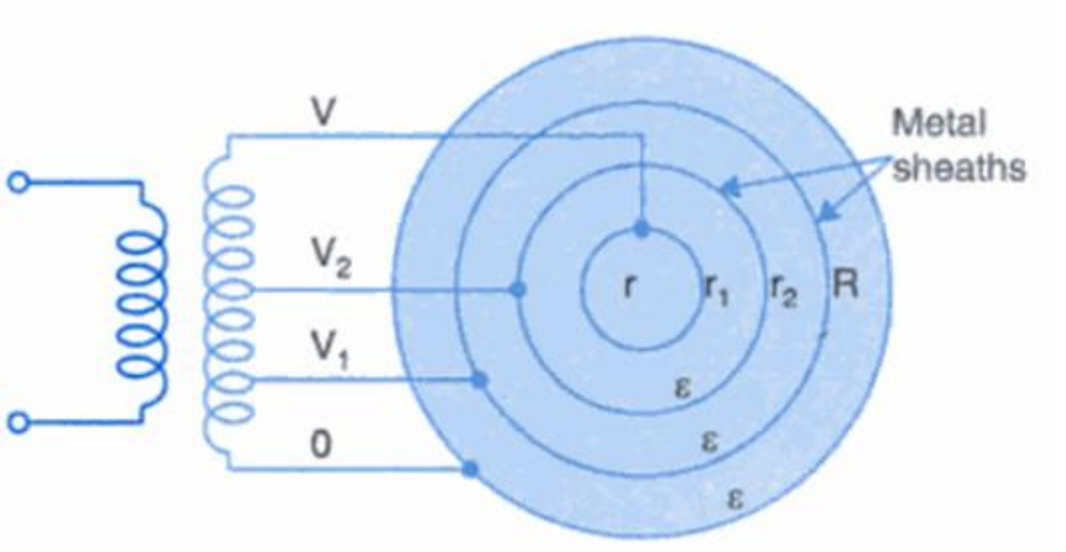


Electrostatic Fields and Field Stress Control ‘Grading of high voltage cables, intersheath grading’

- The voltage of the cable with this intersheath arrangement is given by

$$V = g_{max}\left[r\ln\left(\frac{r_1}{r}\right)+r_1\ln\left(\frac{r_2}{r_1}\right)+r_2\ln\left(\frac{R}{r_2}\right)\right]$$

$$= g_{max}\ln(\alpha)\left[r+r_1+r_2\right]$$



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Example

- A 66 kV concentric cable with two intersheaths has a core diameter 1.8 cm. Dielectric material 3.5 mm thick constitutes the three zones of insulation. Determine the maximum stress in each of the three layers if 20 kV is maintained across each of the inner two.

Electrostatic Fields and Field Stress Control ‘Grading of high voltage cables, intersheath grading’

$$V - V_1 = g_{1,max} r \ln(r_1 / r) = 0.2956 g_{1,max} = 20$$

$$V_1 - V_2 = g_{2,max} r_1 \ln(r_2 / r_1) = 0.3085 g_{2,max} = 20$$

$$V_2 - 0 = g_{3,max} r_2 \ln(r_3 / r_2) = 0.3165 g_{3,max} = 66 - 40$$

$$g_{1,max} = 67.60 \text{ kV/cm}$$

$$g_{2,max} = 64.83 \text{ kV/cm}$$

$$g_{3,max} = 82 \text{ kV/cm}$$